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NUMERICAL STUDY OF THE NON-ISOTERMAL FLOW OF THE POLYMER MELT WITH UNDERMELTED GRANULES IN THE CONICAL ANNULAR CHANNEL OF A DISC EXTRUDER

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ЧИСЕЛЬНЕ ДОСЛІДЖЕННЯ НЕІЗОТЕРМІЧНОЇ ТЕЧІЇ РОЗПЛАВУ ПОЛІМЕРУ З ДОПЛАВЛЕННЯМ ГРАНУЛ У КОНУСНОМУ КІЛЬЦЕВОМУ КАНАЛІ ДИСКОВОГО ЕКСТРУДЕРА

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Abstract. *The aim of this study is to determine the regularities of the non-isothermal flow process of the melt in a conical annular channel, taking into account the heating and melting of granules due to dissipation energy. A higher rotation frequency contributes to intensive heating of the melt and a more pronounced reduction in granule volume. It has been shown that granules are melted in the conical channel. Tangential velocity components and shear velocities decrease as the cone radius diminishes, and the effective viscosity of the melt depends on temperature and shear rate. Factors affecting the pressure drop in the channel are presented. The results of the work are depicted in graphs at disk speeds of 120 and 150 rpm. The same calculation procedure is applied for the conical annular channel as for the cylindrical annular channel. The only difference is that for the cylindrical annular channel, the longitudinal coordinate* z *is used, whereas for the conical annular channel, the longitudinal coordinate* χ *is used. Thus, the introduction of orthogonal coordinates significantly simplified the overall calculation procedure for processes occurring in the homogenization zone.*

Keywords: *disk extruder, polymers, conical annular channels, modeling, non-isothermal processes, melting.*

Анотація. *На якість розплаву впливає багато параметрів, таких як тип обладнання, геометрія робочих органів і конфігурація каналів, у яких відбувається течія. Щоб отримати хорошу якість розплаву, потрібно підтримувати режим роботи в заданих межах, контролюючи ключові параметри, такі як температура розплаву, від якої залежать його основні характеристики. Мета цього дослідження полягає в тому, щоб визначити закономірності неізотермічного процесу течії розплаву в конусному кільцевому каналі з урахуванням нагрівання і плавлення гранул за рахунок енергії дисипації. Попередні розрахунки каналів зони гомогенізації дискового екструдера вказують на необхідність урахувати те, що частина енергії витрачається на плавлення гранул. Вища частота обертання сприяє інтенсивному нагріванню розплаву і більш інтенсивному зменшенню об'єму гранул. Показано, що гранули доплавлюються в конусному каналі, після чого все тепло іде на нагрівання розплаву. Тангенціальні складові швидкості і швидкості зсуву зменшуються зі зменшенням радіуса конуса, а ефективна в'язкість розплаву залежить від температури та швидкості зсуву. Перепад тиску в каналі визначається рядом факторів, серед яких ширина каналу, коефіцієнт консистентності та показник степеня. Надано графічні залежності розподілу температури розплаву, питомого об'єму гранул, тиску, усереднених тангенціальних швидкостей зсуву та зміни ефективної в'язкості розплаву по довжині* *каналу при швидкостях диска 120 і 150 об/хв. Для конусного кільцевого каналу застосовується така сама процедура розрахунку, як і для циліндричного кільцевого каналу. Різниця лише в тому, що для циліндричного кільцевого каналу застосовується подовжня координата z, а для конусного кільцевого каналу застосовується подовжня координата χ. Тому введення ортогональних координат істотно спростило загальну процедуру розрахунків процесів, які відбуваються в зоні гомогенізації.*

Ключові слова: *дисковий екструдер, полімери, конусні кільцеві канали, моделювання, неізотермічні процеси, плавлення.*

Introduction. The melt quality depends on many factors [1, 2]. A significant factor is the type of equipment and the specifics of the working parts' geometry. A well-chosen screw clearance can lead to improved mixing quality [3]. To achieve high melt quality, it is essential to ensure a stable extrusion process. Therefore, it is crucial to understand the flow nature and the primary extrusion process parameters that significantly influence the process itself. Such parameters include, for example, the melt temperature, which varies throughout the homogenization zone and on which the thermophysical and rheological characteristics depend.

Literature review and problem statement. Preliminary calculations have shown that if all granules are fully melted at the entrance to the homogenization zone, the melt temperature rises sharply and significantly deviates from actual readings. This suggests the need to consider energy consumption for the additional melting of granules in the melt. This is confirmed in study [4], where samples obtained after stopping the melting were analyzed. It can be inferred that the solid polymer, in a fully filled area, is displaced by the melt to the upper part of the clearance, and melting occurs due to energy dissipation in the melt films. Experiments have shown that the most stable operating modes are those in which 50–60 % of the melt forms in the screw thread [4]. The homogenization zone of the disc extruder consists of 4 channels (Fig. 1), which includes two straight annular channels, a conical annular channel, and a disc clearance. The straight annular has been discussed in works [5, 6], and in work [7], a computational experiment of the melt flow model with undermelted granules was conducted, describing the calculation methodology. It also demonstrates that the granules continue to melt in the conical channel. In work [8], the flow processes in the conical channel and the calculation procedure in analytical form are described for the first time. This article addresses the flow processes in the conical channel with undermelted granules.

The aim and objectives of the study. The objective of the study is to investigate the non-isothermal flow process of the melt in the conical annular channel, which contains solid particles, to describe the heating and melting process of granules due to dissipation energy. In line with this objective, the following tasks have been set:

- Determine the regularities of the non-isothermal flow process of the melt in the conical annular channel;
- Identify the change in pressure, average melt temperature, average volumetric temperature of the granules, effective viscosity of the melt, and volumetric fraction of granules along the channel length;
- Analyze how the melt temperature in the channel changes, considering the melting of the granules and the disc rotation rate.

The main part of the study. Let a stationary flow of polymer melt occur through a narrow conical annular clearance of width h_K and length L_2, due to a pressure drop ΔP_2, in the χ direction with a constant specified volumetric flow rate (Fig. 2). The generatrix of the cone is at an angle α to the axial line of symmetry O-O_3. The flow of the fluid is considered in the orthogonal coordinate system (χ, κ, θ), which is related to the cylindrical coordinates by the following relations: the axial coordinates of both systems are related by the relationship $\chi = z/\cos\alpha$, and the radial coordinates κ and r are related by the relationship $\kappa = r/\cos\alpha$ [8]. As can be seen in Fig. 2, at a certain distance χ from the beginning of the channel, the radius $R\chi_1$ is the length of the segment AB, and the length of the segment AC is the radius $R\chi_2$. The radii $R\chi_1$ and $R\chi_2$ along the channel length are related by the following relationship:

$$R\chi_2(\chi) = R\chi_1(\chi) + h_\kappa . \quad (1)$$

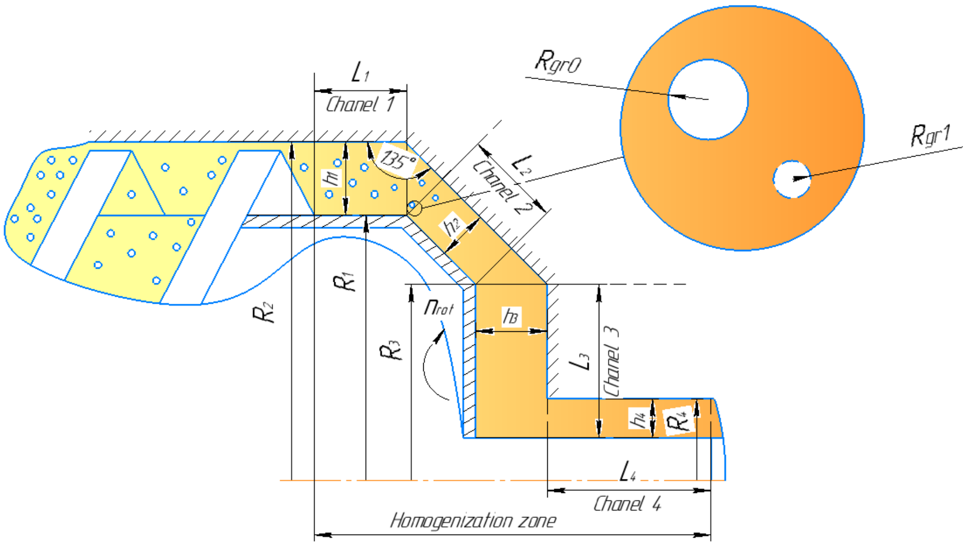


Fig. 1. Schematic image of the homogenization zones of the disk extruder

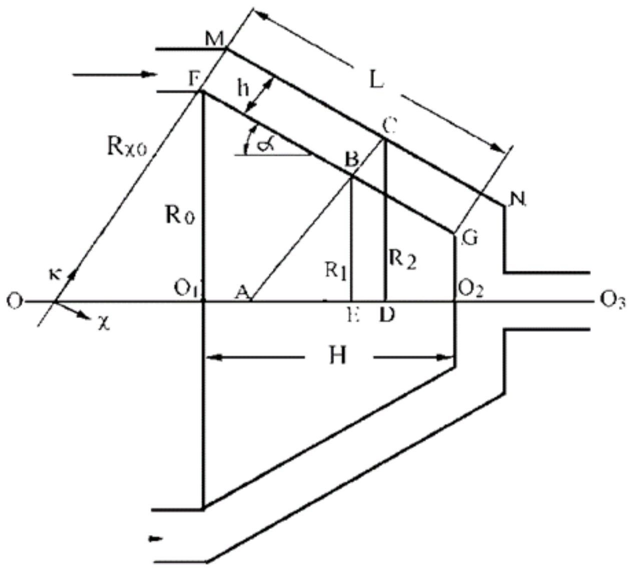


Fig. 2. Schematic representation of the conical annular clearance.
Connection between cylindrical and orthogonal coordinates

Along the coordinate χ radii $R\chi_1$ and $R\chi_2$ in the orthogonal coordinate system depend on R_1 in cylindrical coordinates as follows:

$$R\chi_1 = R_1/\cos\alpha \text{ and } R\chi_2 = R_1/\cos\alpha + h_\kappa . \tag{2}$$

The dependencies of these radii on the coordinate χ are defined by the equations:

$$R_{\chi 1}(\chi) = R\chi_0 - \chi \cdot \mathrm{tg}\,\alpha ; \tag{3a}$$

$$R_{\chi 2}(\chi) = R\chi_0 - \chi \cdot \mathrm{tg}\,\alpha + h . \tag{3б}$$

The radial coordinate ψ has been introduced [5, 8]. It determines the distance of a specified point inside the clearance from the surface of the moving cone and is related to the radial cylindrical coordinate r by the relation:

$$\psi = (r - R_1)/\cos\alpha . \tag{4}$$

The cross-sectional area $S_h(\chi)$ at distance χ is defined as:

$$S_h(\chi) = 2\pi h_\kappa \cdot (R\chi_0 - \chi \mathrm{tg}\,\alpha + h_\kappa/2) \cdot \cos\alpha . \tag{5}$$

The difference from modeling the flow of the non-isothermal process in the first zone lies in the fact that the cross-sectional area $S_h = f(\chi)$ and the average velocity $\bar{v}_\chi = f(\chi)$ vary along the length of the conical channel. Therefore, the width-averaged longitudinal velocity is defined as:

$$\bar{v}_\chi = \frac{G_V}{S_h(\chi)} = \frac{G_V}{2\pi h_\kappa \cdot (R\chi_0 - \chi \mathrm{tg}\,\alpha + h_\kappa/2) \cdot \cos\alpha} . \tag{6}$$

In this problem α =45° so subsequent equations take into account that $\mathrm{tg}\,\alpha = 1$.

The polymer melt, representing a non-Newtonian pseudo-plastic fluid, contains a certain number of monodisperse unfused polymer granules with an initial radius R_{gr0} (Fig. 1). These granules are uniformly distributed throughout the melt volume. The number of granules per unit volume of the mixture remains constant $n_{gr} = \mathrm{const}$. Hence, the initial volumetric fraction of the polymer granules is defined as $V_{gr0} = 4/3\pi R_{gr0}^3 \cdot n_{gr}$ and is measured in m³/m³. The temperature at which the polymer fully transits from solid to liquid state is known and equals T_m. The melt temperature at the channel inlet equals T_{l0}. The initial average volumetric temperature of all granules is uniform and equals $\bar{T}_{gr0} < T_m < T_{l0}$. It is assumed that at temperatures $T \le \bar{T}_{gr0}$, the polymer is in solid state, while at temperatures $T \ge T_m$, it's in liquid state. Within the temperature range $\bar{T}_{gr0} \div T_m$ the polymer melts, transiting from solid to liquid state. Specified are the density value ρ_l, thermal conductivity λ_l, and specific heat of the melt c_l, which are considered constant at temperatures $T \geq T_m$. Also defined are the density ρ_s and thermal conductivity of the solid phase λ_s, which remain constant within the temperature range $Tgr \div T_m$. In the polymer melting temperature range $T_{gr} \div \overline{T}_m$, the heat capacity initially increases from $c_s(T_{gr0})$ value, and then, after reaching its maximum, decreases to c_l. For an approximate determination of the enthalpy change during polymer melting, an average value $\overline{c}_s$ is used. This value was identified based on studies of data from tables and literature concerning thermal and rheological characteristics of LDPE. It is assumed that, within the studied temperature range, the density of the solid phase of the granules is ρ_s, to an accuracy of 5 %, equal to the density of the polymer melt ρ_l. When the inner cone rotates in viscous fluid, frictional forces arise between adjacent cylindrical melt layers. This friction results in the conversion of mechanical energy into heat, causing the melt to heat up due to viscous energy dissipation. Without loss of generality, we'll assume that the temperature of the channel wall's surface at each clearance section χ equals the current melt temperature $T_l(\chi)$, meaning the process occurs in an adiabatic mode. During the melting process of granules, their volumetric fraction V_{gr} decreases, and correspondingly, the volumetric fraction of the melt $(1-V_{gr})$ increases.

It is necessary to determine the pressure change $p_\chi = f(\chi)$, the average melt temperature $\overline{T}_l = f(\chi)$, the effective melt viscosity $\mu_{ef} = f(\chi)$, the average volumetric granule temperature $\overline{T}_{gr} = f(\chi)$, and the volumetric fraction of granules $V_{gr} = f(\chi)$ along the channel length. The problem is solved in orthogonal coordinates (χ, ψ, θ) [8].

Modeling heat exchange processes in the channel of II zone. Let's assume that the polymer melt, along with a certain amount of unfused granules, enters the conical annular channel of the second zone from the exit of the straight annular channel of the first homogenization zone. We will apply the same assumptions as for the first zone. All granules are monodisperse and have a spherical shape, which they retain until their final melting. The volume of one granule is $V_{gr1} = 4/3 \cdot \pi R_{gr}^3$, where $R_{gr} = f(\tau)$ is the current granule radius, which changes along the channel (Fig. 1). The number of granules per unit volume of melt n_{gr} remains constant throughout the melting process.

Then, the heat energy inflow with the liquid mixture through the conical annular surface with coordinate χ i.e. the enthalpy value at the channel entrance is:

$$H_\chi = \rho_l c_l \bar{v}_\chi (T_l - T_{st})_\chi (1 - V_{gr})_\chi (S_h)_\chi + \rho_s c_s \bar{v}_\chi (\overline{T}_{gr} - T_{st})_\chi n_{gr} V_{gr1}\big|_\chi (S_h)_\chi . \quad (7)$$

The heat energy (enthalpy) outflow with the liquid mixture through the flat annular surface with coordinate $\chi + \Delta\chi$:

$$H_{\chi+\Delta\chi} = \rho_l c_l \bar{v}_{\chi+\Delta\chi} (T - T_{st})_{\chi+\Delta\chi} (1 - V_{gr})_{\chi+\Delta\chi} (S_h)_{\chi+\Delta\chi} + (S_h)_{\chi+\Delta\chi} +$$
$$+ \rho_s c_s \bar{v}_{\chi+\Delta\chi} (\overline{T}_{gr} - T_{s)})_{\chi+\Delta\chi} n_{gr} V_{gr1}\big|_{\chi+\Delta\chi} . \quad (8)$$

Subtracting the first equation (7) from the second (8), we get:

$$\Delta H = \rho_l c_l \cdot \bar{v}_\chi(\chi) S_h(\chi) \cdot \left[\Delta T - \Delta\left(T \cdot V_{gr}\right) + \frac{\rho_s c_s}{\rho_l c_l} \Delta\left(\bar{T}_{gr} \cdot V_g\right) \right]. \qquad (9)$$

It is considered that in any channel section, the product $\bar{v}_\chi(S)_\chi = G_V = \text{const}$ remains constant. Since there are no external heat sources, the enthalpy remains unchanged, so:

$$\Delta H = \rho_l \cdot c_l \cdot \bar{v}(\chi) \cdot S_h(\chi) \cdot \left[\Delta T - \Delta\left(T \cdot V_{gr}\right) + \frac{\rho_s c_s}{\rho_l c_l} \Delta\left(\bar{T}_{gr} \cdot V_{gr}\right) \right] = 0. \qquad (10)$$

We rewrite equation (10), expanding the terms in the round brackets.

$$\Delta H = \rho_l \cdot c_l \cdot \bar{v}_\chi(\chi) \cdot S_h(\chi) \cdot \left[\Delta T - n_{gr} \cdot \Delta T \cdot V_{gr1} - n_{gr} \cdot T \cdot \Delta V_{gr1} + \frac{\rho_s c_s}{\rho_l c_l} \Delta \bar{T}_{gr} \cdot V_{gr} + \frac{\rho_s c_s}{\rho_l c_l} \bar{T}_{gr} \cdot \Delta V_{gr} \right] = 0. \qquad (11)$$

The volume change of the granule $\Delta V_{gr1} = 0$ occurs because a portion of the granule volume melts and transitions from a solid state to a liquid one i.e. a phase transition takes place. This requires a quantity of heat (enthalpy) equal to $Q_{melt} = n_{gr} \Delta V_{gr1} \cdot \bar{c}_s \left(T_m - \bar{T}gr\right)$. This is the latent heat of polymer melting, defined as the increase in temperature of the polymer's solid phase. The heat of melting, i.e., the increase in enthalpy of the solid phase, is spent from the enthalpy of the melt itself. Moreover, at each step, a portion of the melt's enthalpy is spent on heating the melted portion of the polymer from the temperature T_m to the current melt temperature $T(\chi)$. This part of the melt's enthalpy change equals $Q_{нагр} = n_{gr} \Delta V_{gr1} \cdot \bar{c}_s \left(T(\chi) - T_m\right)$. As a result, the enthalpy change ($Q_{melt} + Q_{нагр}$) is compensated by the change in melt's enthalpy so that the overall enthalpy of the «melt-granule» system remains unchanged ($\Delta H = 0$), provided there are no external heat sources.

Considering the latent heat of melting, the right side of the heat balance equation (11) is written as:

$$\bar{v}(\chi) S(\chi) \cdot \left\{ \rho_l c_l \left(1 - V_{gr}\right) \cdot \Delta T - \left[\rho_l c_l \cdot \left(T_l - Tm\right) + \rho_s \bar{c}_s \left(T_m - \bar{T}_{gr}\right) \right] \cdot \Delta V_{gr} + \rho_s c_s \cdot \Delta \bar{T}_{gr} \cdot V_{gr1} \right\} = 0. \qquad (12)$$

In view of viscous dissipation in the heat balance equation, it's necessary to account for the fact that the liquid melt occupies only a portion of the volume, equal to $\left(1 - V_{gr}\right)$. Thus, the dissipation power, which indicates the amount of heat transferred to the layer $d\chi$ with volume $S_h d\chi$ in a unit of time, is described by the equation:

$$\Delta H_{dis} = \mu_{ef}(T) \cdot \left(\bar{\dot{\gamma}}_{\psi\chi}^2 + \bar{\dot{\gamma}}_{\psi\theta}^2 \right) \cdot \left(1 - V_{gr}\right) \cdot S_h \Delta\chi, \qquad (13)$$

or its equivalent equation:

$$\Delta H_{dis} = K_T \cdot \left[\left(\bar{\dot{\gamma}}_{\psi\chi}^2 + \bar{\dot{\gamma}}_{\psi\theta}^2\right)\right]^{\frac{1+n}{2}} \cdot \left(1 - V_{gr}\right) \cdot S_h \Delta\chi \,. \quad (14)$$

Where $\mu_{ef}(T)$ is the effective viscosity of the melt, described for pseudoplastic fluids by the Oswald-de Waele power-law; $\bar{\dot{\gamma}}_{\psi\chi} = d\bar{v}_\chi / d\psi$ is the average shear rate of the longitudinal flow across the clearance width; $\bar{\dot{\gamma}}_{\psi\chi} = d\bar{v}_\chi / d\psi$ is the average shear rate of the tangential flow across the clearance width; $K_T = f(T)$ is the consistency coefficient; $n = f(T)$ is the flow index.

As in previous works [5, 8], the shear rate of longitudinal flow $\dot{\gamma}_{\psi\chi}$ is negligibly smaller than the shear rate of tangential flow $\dot{\gamma}_{\psi\theta}$ and can be disregarded, which is applied in subsequent calculations. The average shear rate of the tangential flow in the conical annular channel $\bar{\dot{\gamma}}_{\psi\theta}$ is determined from the equation:

$$\bar{\dot{\gamma}}_{\psi\theta}(\chi) = -\frac{2\omega_0}{\left[\left(\frac{\left(R_{\chi 1} + h_\kappa/2\right)}{R\chi_1 - h_\kappa/2}\right)^2 - \left(\frac{\left(R_{\chi 1} + h_\kappa/2\right)}{R\chi_1 + 3h_\kappa/2\cdot}\right)^2\right]}. \quad (15)$$

Equation (15), considering equation (3a), describes the change in the average shear rate along the channel length.

The amount of enthalpy entering the layer $\Delta\chi$ due to viscous dissipation can be represented through effective viscosity by the equation:

$$\Delta H_{dis} = \mu_{ef}(T) \cdot \dot{\gamma}_{\psi\theta}^2 \cdot \left(1 - V_{gr}\right) \cdot S_h \Delta\chi \quad (16)$$

or apply the consistency coefficient in the form of an equivalent equation:

$$\Delta H_{dis} = K_T \cdot \dot{\gamma}_{\psi\theta}^{1+n} \cdot \left(1 - V_{gr}\right) \cdot S_h \Delta\chi \,. \quad (17)$$

Thus, the enthalpy difference of the mixture $\Delta H_{\Delta\chi}$ at any section of the channel is only due to the heat of viscous dissipation and equals the dissipation power q_{dis} on that section. Taking into account equations (12) and (17), we will write the heat balance equation in layer $\Delta\chi$ in the presence of thermal dissipation:

$$\bar{v}(\chi) \cdot S(\chi) \cdot \left\{\rho_l c_l \cdot \left(1 - V_{gr}\right) \cdot \Delta T - \left[\rho_l c_l \cdot \left(T_l - Tm\right) + \rho_s \bar{c}_s \cdot \left(T_m - \bar{T}_{gr}\right)\right] \cdot \Delta V_{gr} + \right.$$
$$\left. + \rho_s c_s \cdot \Delta\bar{T}_{gr} \cdot V_{gr1}\right\} = K_T \cdot \dot{\gamma}_{\psi\theta}^{1+n} \cdot \left(1 - V_{gr}\right) \cdot S_h \Delta\chi \,. \quad (18)$$

Dividing both parts of equation (12) by $S(\chi)\Delta\chi$ and transitioning to the limit $\Delta\chi \to 0$:

$$\bar{v}(\chi)\left\{\rho_l c_l\left(1-n_{gr}V_{gr1}\right)\frac{dT}{d\chi}-\left[\rho_l c_l\left(T_l-Tm\right)+\rho_s\bar{c}_s\left(T_m-\bar{T}_{gr}\right)\right]n_{gr}\frac{dV_{gr1}}{d\chi}+\right.$$

$$\left.+\rho_s c_s\frac{d\bar{T}_{gr}}{d\chi}n_{gr}V_{gr1}\right\}=K_T\dot{\gamma}_{\psi\theta}^{1+n}\left(1-V_{gr}\right). \qquad (19)$$

It follows that the temperature change of the melt along the length of the channel of the second zone can be represented in the form of the equation:

$$\frac{dT}{d\chi}=\frac{1}{\left(1-V_{gr}\right)}\left\{\frac{K_T\dot{\gamma}_{\psi\theta}^{1+n}\cdot\left(1-V_{gr}\right)}{\rho_l c_l\cdot\bar{v}(\chi)}+\left[\left(T_l-Tm\right)+\frac{\rho_s\bar{c}_s}{\rho_l c_l}\left(T_m-\bar{T}_{gr}\right)\right]\frac{dV_{gr}}{d\chi}-\frac{\rho_s\bar{c}_s}{\rho_l c_l}\frac{d\bar{T}_{gr}}{d\chi}\cdot V_{gr}\right\}. \qquad (20)$$

The rate of volume change of granules $dV_{gr}/d\chi$ during their melting is a negative value, so the second term in curly brackets is also negative. Equation (19) contains three unknown variables that depend on the coordinate χ. These are the melt temperature T, the granule volume V_{gr}, and the average volume temperature of the granule $\bar{T}_{gr}$. To determine the latter two parameters, two additional equations are required that describe the relationships $V_{gr}=f(\chi)$ and $\bar{T}_{gr}=f(\chi)$.

Calculation of the granule melting rate. A polymer mixture at temperature $T_l(\chi)$ passes through a channel and, from the melt to granules with surface temperature T_m, heat is transferred. Due to this heat, the surface layer of the granules melts and they heat up. This means their average temperature increases $\bar{T}_{gr}$. The melting rate of a single granule can be denoted as $dm_{gr1}/d\tau=\rho_{gr}\,dV_{gr1}/d\tau$. Then, for a set of monodisperse granules, the melting rate can be determined from the heat exchange equation. The heat exchange equation for granules with the melt can be represented as follows:

$$4\pi R_{gr1}\lambda_l\left(T_l-T_m\right)\cdot n_{gr}=4\pi R_{gr1}^2\frac{dR_{gr1}}{d\chi}n_{gr}\rho_l\bar{c}_s\left(T_m-\bar{T}_{gr}\right)\cdot\bar{v}_\chi+4\pi R_{gr1}\lambda_s\left(T_m-\bar{T}_{gr}\right)\cdot n_{gr}. \qquad (21)$$

The left side of this equation is the amount of heat transferred from the melt to the granules through heat conduction. The first term on the right side of (21) represents the amount of introduced heat used to melt the granule's surface layer. The second term on the right side represents the amount of introduced heat used for heating the granules. The heat exchange equation is expressed in terms of the total volume of the granule aggregate, rather than the radius of the granule, using the following relationships:

$$4\pi R_{gr1} = \left(\frac{3V_{gr}}{4\pi n_{gr}}\right)^{0,33} = 7.8 \cdot \left(\frac{V_{gr}}{n_{gr}}\right)^{0,33} \quad \text{and} \quad 4\pi R_{gr1}^{2}\frac{dR_{gr1}}{d\chi}n_{gr} = \frac{dV_{gr}}{d\chi}. \qquad (22)$$

Thus, the heat exchange equation is reduced to:

$$\rho_l \cdot c_s\left(T_m - \overline{T}_{gr}\right)\overline{v}_\chi \frac{dV_{gr}}{d\chi} = 7,8 \cdot \left(\frac{V_{gr}}{n_{gr}}\right)^{0,33} n_{gr}\left[\lambda_l\left(T_l - T_m\right) - \lambda_s\left(T_m - \overline{T}_{gr}\right)\right]. \qquad (23)$$

Solving this equation in terms of $dV_{gr}/d\chi$, we get a differential equation for the granule melting rate in the melt flow per unit length of the channel.

$$\frac{dV_{gr}}{d\chi} = -\frac{7,8 \cdot \left(V_{gr}/n_{gr}\right)^{0,33} \cdot n_{gr}}{\rho_l \cdot c_s\left(T_m - \overline{T}_{gr}\right) \cdot \overline{v}_\chi} \cdot \left[\lambda_l\left(T_l - \overline{T}_m\right) - \lambda_s\left(T_m - \overline{T}_{gr}\right)\right]. \qquad (24)$$

The obtained differential equation describes the dependency $V_{gr} = f(\chi)$.

Calculation of the change in the average volume temperature of the granule. As seen from equation (20), out of all the heat transferred from the melt to the granules over a section of the channel $d\chi$, a portion of the heat is used to increase their average volume temperature $\overline{T}_{gr}$:

$$4\pi R_{gr1}\left(T_m - \overline{T}_{gr}\right) \cdot n_{gr} = \rho_l \overline{c}_s \overline{v}_\chi \left(\frac{4}{3}\pi R_{gr1}^3\right)\frac{d\overline{T}_{gr}}{d\chi} n_{gr}. \qquad (25)$$

By simplifying both parts of equation (25), we get:

$$\frac{d\overline{T}_{m1}}{d\chi} = \frac{3\lambda_s \cdot \left(T_m - \overline{T}_{gr}\right)}{\rho_l \overline{c}_s \cdot \overline{v}_\chi \cdot R_{gr1}^2}. \qquad (26)$$

Substituting relationship (22) into (26), we bring the equation to the form:

$$\frac{d\overline{T}_{gr}}{d\chi} = \frac{3\lambda_s \cdot \left(T_m - \overline{T}_{gr}\right)}{0,384 \cdot \left(V_{gr}/n_{gr}\right)^{0,66} \cdot \rho_l \cdot \overline{c}_s \cdot \overline{v}_\chi \cdot}. \qquad (27)$$

As a result, a differential equation for the relationship $\overline{T}_{gr} = f(\chi)$ is obtained.

The system of equations for the heating and melting processes in II zone includes differential equations and algebraic equations.

1. The melt temperature equation, expressed through the consistency coefficient:

$$\frac{dT}{d\chi} = \frac{1}{\left(1-V_{gr}\right)\cdot c_l}\left\{\frac{K_T\cdot\left(\bar{\dot{\gamma}}_{\psi\theta}\right)^{1+n}\left(1-V_{gr}\right)}{\rho_l\cdot v_z} + \left[c_l\left(T-T_m\right)+c_s\left(T_m-T_{gr}\right)\right]\frac{dV_{gr}}{d\chi} - c_s V_{gr}\frac{dT_{gr}}{d\chi}\right\}. \quad (28)$$

Boundary condition: $T_l(0) = T_0$.

2. Equation for the change in granule volume in the mixture:

$$\frac{dV_{gr}}{d\chi} = -\frac{7{,}8\cdot\left(V_{gr}/n_{gr}\right)^{0{,}33} n_{gr}}{\rho_l\cdot c_s\left(T_m-\bar{T}_{gr}\right)\cdot\bar{v}_\chi}\cdot\left[\lambda_l\left(T_l-\bar{T}_m\right)-\lambda_s\left(T_m-\bar{T}_{gr}\right)\right]. \quad (29)$$

Boundary condition: $V_{gr}(0) = V_{gr0}$.

3. Equation for the average temperature of the granule:

$$\frac{dT_{gr}}{d\chi} = \frac{3\lambda_s\cdot\left(T_m-T_{gr}\right)}{0{,}384\cdot\left(V_{gr}/n_{gr}\right)^{0{,}66}\rho_l c_s v_\chi\cdot}. \quad (30)$$

Boundary condition: $T_{gr}(0) = T_{gr0}$.

4. Equation for the pressure gradient in the channel:

$$\frac{dp}{d\chi} = \frac{A_2}{\left(2R\chi_0 + h_\kappa - 2\chi\right)^n}, \quad (31)$$

where
$$A_2 = -\frac{K_T\cdot\left(G_V\right)^n}{\left(h_\kappa/2\right)^{2n+1}}\cdot\left(\frac{2n+1}{2\pi\cdot n\cdot\cos\alpha}\right)^n.$$

Equation (31) is solved with the boundary condition: $p(0) = p_0$.

In addition to differential equations, the model also includes algebraic equations describing:

1. The change in the cross-sectional area along the length of the channel:

$$S_h(\chi) = 2\pi h_\kappa\cdot\left(R\chi_0 - \chi\,\mathrm{tg}\,\alpha + h_\kappa/2\right)\cdot\cos\alpha. \quad (32)$$

2. The change in average longitudinal velocity along the length of the channel:

$$\bar{v}_\chi = G_V/S_h(\chi). \quad (33)$$

3. The change in the average clearance width shear velocity of tangential movement along the channel length χ:

$$\bar{\dot{\gamma}}_{\psi\theta}(\chi) = -\frac{2\omega_0}{\left[\left(\frac{\left(R_{\chi 0} - \chi + h_\kappa/2\right)}{R\chi_0 - \chi - h_\kappa/2}\right)^2 - \left(\frac{\left(R_{\chi 0} - \chi + h_\kappa/2\right)}{R\chi_0 - \chi + 3h_\kappa/2\cdot}\right)^2\right]}. \quad (34)$$

4. The dependence of the consistency coefficient K_T on temperature:

$$K_T(T) = K_{st} \cdot \exp\left[-\beta \cdot (T_l - T_{st})\right], \quad (35)$$

where $K_{st} = K_T(T_{st})$; β is viscosity temperature coefficient.

5. The dependence of the exponent on temperature:

$$n = n_{st} + \alpha_n (T_l - T_{st}), \quad (36)$$

where $n_{st} = n(T_{st})$, α_n are temperature coefficients.

6. Dependence of effective viscosity on temperature:

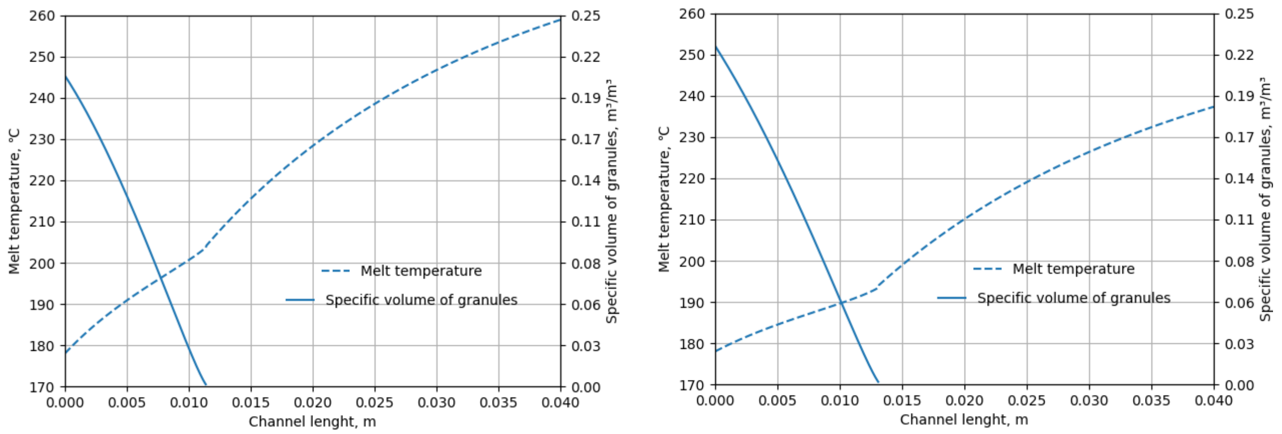
$$\mu_{ef}(T) = K_T(T) \cdot \left(\gamma_{\psi\theta}\right)^{n-1}. \quad (37)$$

The obtained differential equations for the conical annular channel are analogous in form to the corresponding equations of the straight cylindrical annular channel. The difference lies in that instead of the longitudinal coordinate z, the longitudinal coordinate χ is considered in the second channel. The introduction of orthogonal coordinates significantly simplifies the calculation procedure for processes in this homogenization zone. Moreover, the equation system for the conical annular channel is supplemented by three algebraic equations (equations (33), (34) and (35)) that account for changes in the channel cross-sectional area and disc radius, as well as the associated change in shear velocities of longitudinal and tangential movement.

Analysis of Research Results. A flow model of the melt with unfused granules in the conical annular channel has been created and a computational experiment was conducted. The length of the straight annular clearance L_2 = 40 mm, width h_K = 2,5 mm. The experiment was performed at two rotational speeds of the screw ω_0 – 120 rpm and 150 rpm. High-pressure polyethylene of grade 15803-020 was chosen as the model object.

The melt with granules moves through the conical annular channel at a constant volumetric flow rate of G_V =9,06·10**-6** m³/s with an average speed $\bar{v}_\chi(\chi)$ that increases in length due to a reduction in the cross-sectional area. Changes in the cross-sectional area $S_h = f(\chi)$ and the cone radius along the channel length significantly influence the changes in temperature, pressure, viscosity, tangential, and longitudinal speed and the corresponding shear rates. As shown above, since the shear rate of longitudinal flow can be neglected when calculating the intensity of dissipation, the degree of dissipative heating of the melt depends solely on the shear rate $\bar{\dot{\gamma}}_{\psi\theta}$ and thus decreases in length.

According to equations (28) and (29), the rate of temperature change of the melt and the melting rate of the granules along the channel length should depend on the rotation frequency of the inner cone. As can be seen in (Fig. 3, a, b), the higher the rotation frequency of the inner cone, the more the melt temperature increases.



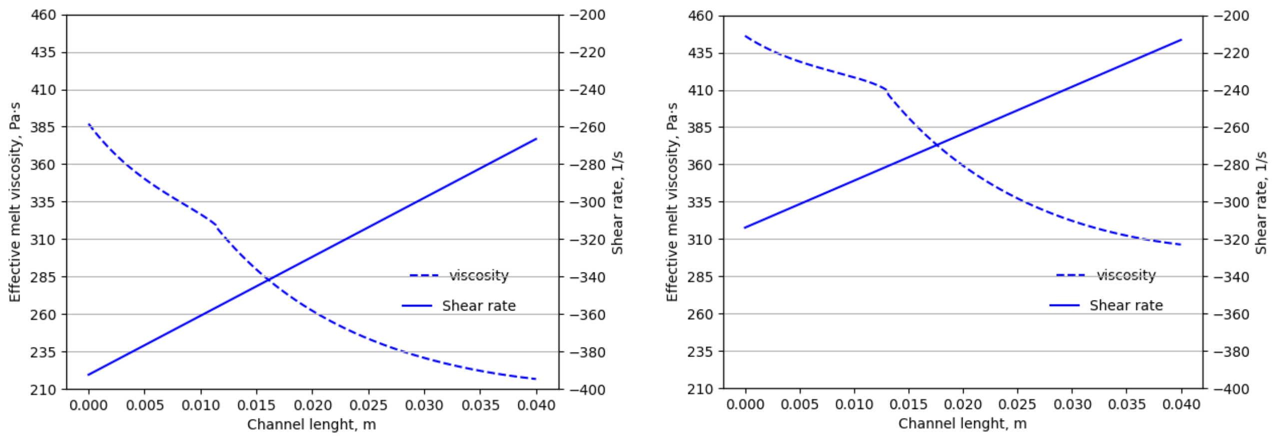
a *b*

Fig. 3. Distribution of the melt temperature T and the specific volume of granules V_{gr} along the length of the II zone at different rotation frequencies:*a* – ω_0 =150 rpm; *b* – ω_0 =120 rpm

The figures show that as the temperature increases more intensively, the melting process of the granules becomes more intense and their total volume decreases faster. Also, in (Fig. 3, a, b), it can be observed that the granules completely melt inside the conical annular channel and the subsequent heat of dissipation is exclusively used for heating the melt.

According to the increase in melt temperature along the length of the channel, the effective viscosity of the melt decreases, as illustrated by the data presented in (Fig. 4, a, b).

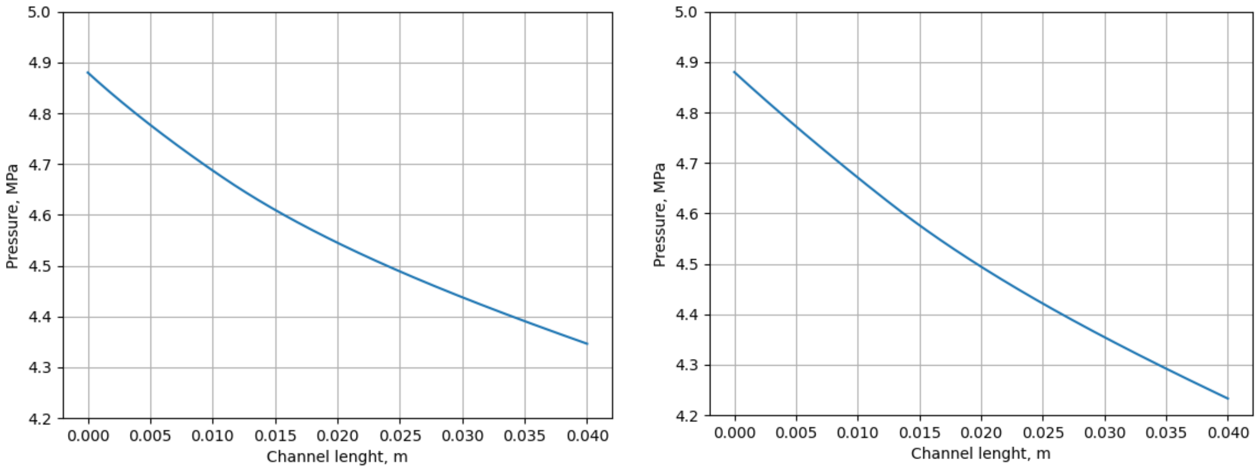
As can be seen from the figures, the change in the screw's rotation number significantly affects the change in the melt's viscosity. Fig. 4, a, b also show that due to the decrease in the cone's radius with the channel length, the average tangential shear speed $\bar{\dot{\gamma}}_{\psi\theta}(\chi)$, determined by equation (34), decreases almost linearly along the length. Unlike in the straight annular channel, in the conical annular channel, the tangential shear speed in each channel cross-section χ increases proportionally to the rotation frequency ω_0.



a *b*

Fig. 4. Distribution of the effective melt viscosity and the shear rate of the tangential flow along the length of the II zone at different rotation frequencies: *a* – ω_0 =150 rpm; *b* – ω_0 =120 rpm

As shown in (Fig. 5, a, b), the pressure drop in the channel $\Delta p(L_2)$ is approximately up to 8 atm and, as can be seen from equation (31), depends on the channel's width, the consistency coefficient $K_T = f(\chi)$, and the degree index $n = f(\chi)$, which depend on the melt temperature. The latter, in turn, depends on the rotation frequency of the disk.



a *b*

Рис. 5. Distribution of the pressure along the length of the channel of II zone at different rotation frequencies:
a – ω_0 =150 rpm; *b* – ω_0 =120 rpm

Conclusions. Modeling of heat exchange processes in a conical channel was conducted. The heating and melting processes of granules at disk rotation frequencies of 120 and 150 rpm have been described and calculated. Calculations indicate that the granules are melted in the second channel. It is shown that with an increase in the rotation of the inner cone, the melt temperature rises, and the volume of granules in the mixture decreases faster. The process calculations were carried out in a conical orthogonal coordinate system.

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