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DYNAMICS OF A PULSE-WIDTH CONVERTER USING THE THEORY OF GENERALIZED FUNCTIONS

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ДИНАМІКА ШИРОТНО-ІМПУЛЬСНОГО ПЕРЕТВОРЮВАЧА ІЗ ЗАСТОСУВАННЯМ ТЕОРІЇ УЗАГАЛЬНЕНИХ ФУНКЦІЙ

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Abstract. *The aim of the paper is an accurate description of the dynamic processes of a pulsed DC voltage converter with pulse-width modulation of the second kind in the mode of continuous load current using the theory of generalized functions of one argument for a one-sided and two-sided reference signal and checking the adequacy of the obtained pulse models of the converter by simulating in the MATLAB environment of a high-speed system for regulating the output current of the converter. The novelty of the proposed work consists in the application of the theory of generalized functions of one argument to describe the dynamic processes of a pulse converter of direct voltage with pulse-width modulation of the second kind. The practical value of the work consists in the fact that the application of the analytical dependencies is obtained in the article in the process of developing an automatic control system for the output coordinates of a pulse converter makes it possible to calculate the system parameters for a given dynamic process.*

Keywords: *pulse converter, direct voltage, pulse-width modulation of the second kind, one-sided and two-sided reference signal, generalized derivative, pulse model, transient process.*

Анотація. *Метою роботи є точний опис динамічних процесів імпульсного перетворювача постійної напруги з широтно-імпульсною модуляцією другого роду в режимі безперервного струму навантаження із застосуванням теорії узагальнених функцій одного аргументу для одностороннього і двостороннього опорного сигналу і перевірка адекватності отриманих імпульсних моделей перетворювача за рахунок імітаційного моделювання в середовищі MATLAB швидкодіючої системи регулювання вихідного струму перетворювача. Показано ефективність застосування методу узагальненої похідної для отримання імпульсних моделей імпульсного перетворювача постійної напруги з широтно-імпульсною модуляцією в режимі безперервного струму. Новизна запропонованої роботи полягає в застосуванні теорії узагальнених функцій одного аргументу для опису динамічних процесів імпульсного перетворювача постійної напруги з широтно-імпульсною модуляцією другого роду. Отримані імпульсні моделі дають змогу точно описати динамічні процеси в системах автоматичного регулювання з імпульсним перетворювачем постійної напруги, розрахувати параметри регулятора для отримання граничної швидкодії. Аналіз отриманих осцилограм показує, що перехідний процес закінчується за два тактові інтервали імпульсного перетворювача напруги, що відповідає порядку системи. Аналіз динамічних процесів розглянутої системи підтверджує коректність аналітичних результатів, одержаних під час* *проведення дослідження. Практична цінність роботи полягає в тому, що застосування отриманих у роботі аналітичних залежностей у процесі розроблення системи автоматичного регулювання вихідних координат імпульсного перетворювача дає змогу розрахувати параметри системи заданого динамічного процесу.*

Ключові слова: *імпульсний перетворювач, постійна напруга, широтно-імпульсна модуляція другого роду, односторонній і двосторонній опорний сигнал, узагальнена похідна, імпульсна модель, перехідний процес.*

Introduction. Pulsed DC voltage converter with pulse-width modulation are widely used in automated electromechanical systems [1, 2] and for the construction of stabilized power supply system's [3, 4]. Such converters allow the creation of systems with high dynamic characteristics [5–8]. To implement their properties, it is necessary to use precise mathematical models and a special mathematical apparatus.

Differences in the dynamic processes of impulse converters are caused by different forms of reference signals in control systems [9]. Such converters are abused by the forms of support signals of control systems: in converters from one-sided pulse-width modulation – this is a one-sided saw tooth signal, and in converters with double-sided modulation – this is a double-sided support signal. Also, due to the fact that the output voltage is a sequence of rectangular pulses, at the output of the above continuous part, a variable component is formed, which falls on the input of the control system. The presence of a variable component, differences in the forms of reference signals and the need to strictly describe mathematical models affects the accuracy of calculation of dynamic processes of pulsed DC voltage converter with pulse-width modulation, which is formed in the research of performed in this paper.

Literature review and problem statement. The study of dynamic processes in pulsed buck DC converters is presented in works [10, 11]. References [12–14] provide research results on the dynamic proper-ties of automatic control systems with one-sided pulse-width modulation. There are significantly fewer research paper dedicated to systems with two-sided modulation. It should be noted that the known research results obtained from the study of electro-magnetic processes without strict mathematical descriptions do not provide a complete understanding of the adequacy of the dynamic models used for converters. The complexity of describing dynamic processes arises from the pulsed nature of electrical energy conversion, which leads to the presence of discontinuous functions.

The aim and objectives of the study. The purpose of this work is to study the dynamic models of a pulsed DC voltage converter with pulse-width modulation operating in continuous current mode. To achieve this purpose, the following tasks are set:

– to justify the applicability of the mathematical apparatus of generalized functions for studying the dynamic characteristics of pulsed buck DC converters operating in continuous current mode in the load circuit;

– to investigate the dynamic characteristics of control systems using the obtained pulsed models.

The main part of the study. ***Dynamics of a pulsed dc converter with one-sided modulation.*** The dynamic processes of the pulsed converter are considered for the continuous current mode in the load circuit. The converter is powered by an ideal voltage source, in this case, a sequence of rectangular pulses is formed at its output $e_v = \gamma \cdot U_m$.

Fig. 1 shows a generalized equivalent circuit of a pulse-width converter with pulse-width modulation (PWM).

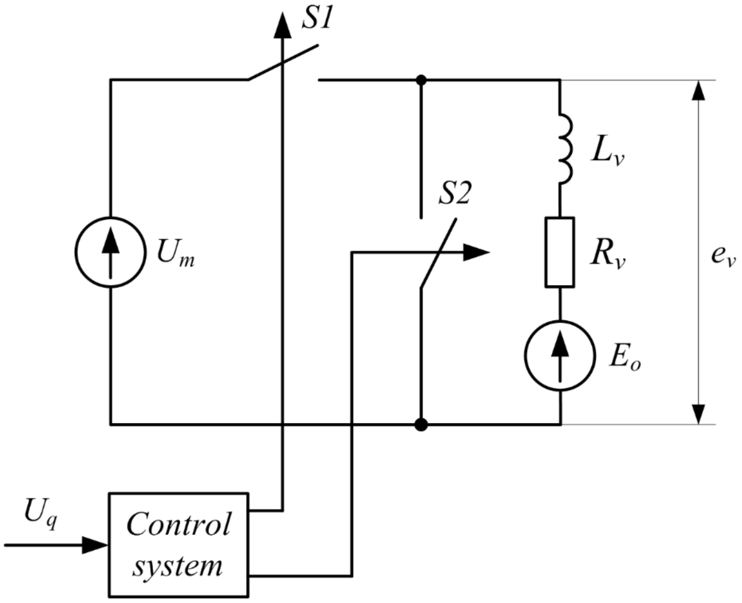

Fig. 1. Generalized equivalent circuit of a pulse-width modulated converter with pulse-width modulation

The process of current flow in the load is described by a system of differential equations:

$$L_v \frac{dI_v^{'}}{dt} + I_v^{'} R_v = U_m - E_o; \quad L_v \frac{dI_v^{''}}{dt} + I_v^{''} R_v = -E_o. \tag{1}$$

The dynamic processes in the converter are determined by the properties of the informational component of the transient process. To obtain this component, a perturbation ΔU_q is applied to the control signal of the converter (Fig. 2). As a result, variations in the output pulses with a duration of $\Delta\gamma_n T$ are observed.

The following notations are used in Fig. 2: T – the switching period of the pulse-width modulated converter; $\gamma_n T$ – the duration of the PWM output pulse; $\Delta\gamma_n T$ – the variation in the duration of the PWM output pulse.

The process of converting the input voltage into a sequence of rectangular pulses is determined by two switching conditions of the power switch. The first switching condition is related to the operating principle of the control system. For a Inertia-free control system implementing the vertical control principle with a one-sided reference signal, this condition is expressed as follows

$$U_q(t_n) = U_s(t_n), \tag{2}$$

where $U_q(t_n)$ – control signal;

$U_s(t_n)$ – PWM reference signal;

t_n – time corresponding to the switching at the n-th PWM interval.

The first switching condition connects the magnitude of the control signal with the duration of the output pulse of the PWM in the interval of energy transfer by the rectifier from the input source to the load and determines the moment of transition from the first equation of the system (1) to the second. The second switching condition is functionally connected with the first and is fulfilled when the load current, during the open state of the power switch S1 (Fig. 1), decreases to zero.

$$I_v(t_n; t_n + \tau) = 0, \tag{3}$$

where τ – time for load current to fall to zero.

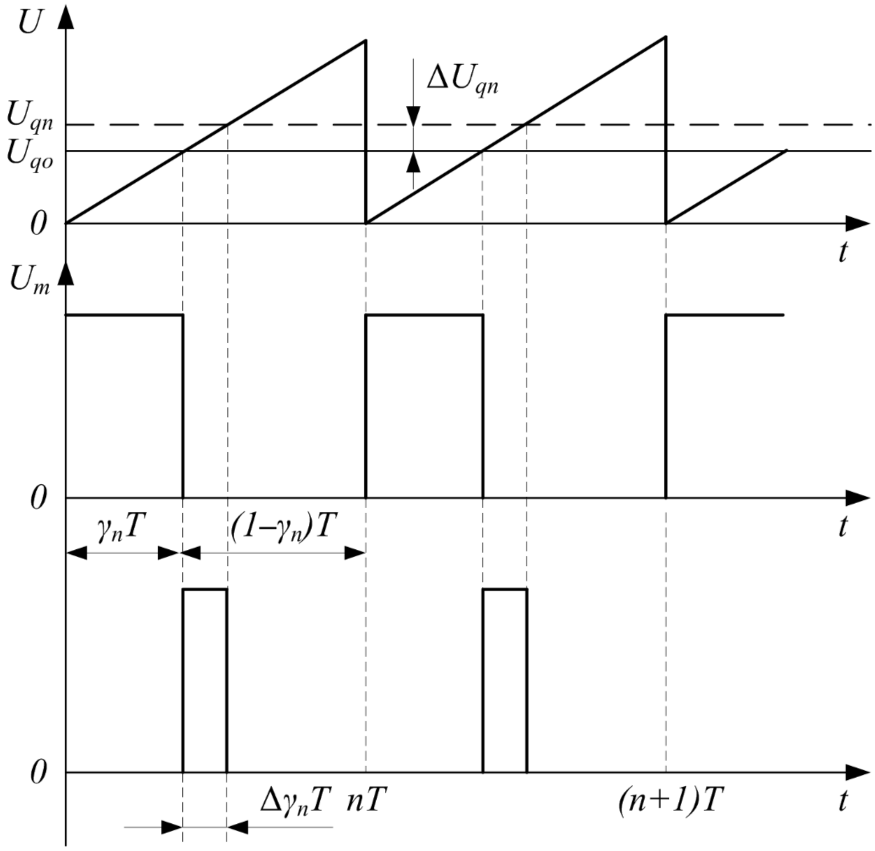

Fig. 2. Voltage diagrams of a pulse-width modulation converter with a one-sided reference signal

Condition (3) is satisfied when the PWM converter switches to the intermittent current mode, when

$$E_o > U_m. \tag{4}$$

The given system of piecewise linear equations (1) – (3) allows to describe electromagnetic and dynamic processes of a pulse converter with PWM.

The output voltage increment under the action of ΔU_q in a closed structure is determined by a transcendental equation with respect to the control signal.

$$\Delta e_v\left[\Delta U_q\left(t_n\right)\right] = e_v\left(U_q + \Delta U_q\right) - e_v\left(U_q\right). \tag{5}$$

The dynamic relationship between the input ΔU_q and output Δe_v coordinates is determined by linearizing equation (5) relative to the new state of the system ΔU_{qn}. Having expanded equation (5) into a generalized Taylor series and limited to the first two terms of the expansion, we obtain

$$\Delta e_v = \frac{De_v\left(t_{\gamma n}\right)}{dU_q} \cdot \frac{\Delta U_q}{1!}, \tag{6}$$

where $\frac{De_v(t_{\gamma n})}{dU_q} = \frac{\partial e_v}{\partial U_q} - \Delta E_n(t_{\gamma n})\frac{dt_{\gamma n}}{dU_q}\cdot\delta(t - t_{\gamma n})$ – generalized derivative of a function that has a discontinuity of the first kind and depends on a parameter [11];

$\Delta E_n(t_{\gamma n}) = U_m$ – the magnitude of the jump in the output electromotive force at the break point at each PWM discreteness interval.

At the intervals of the output pulse action, the output electromotive force is invariant to changes in the controlled parameter *γ*, and therefore

$$\frac{de_v}{\partial U_q} = 0. \tag{7}$$

Taking into account (7), expression (6), written for the *n*-th discrete interval of the pulse-width modulated converter, takes the following form:

$$\Delta e_v(t_n) = -\frac{\Delta U_q(t_n)}{1!}\cdot U_m\cdot\frac{dt_n}{dU_q}\cdot\sum_{n=0}^{k_o}\delta(t - t_n). \tag{8}$$

The relationship between the increments of the control input ΔU_q and the increments of the time coordinate t_n is determined by the solution of a transcendental equation derived from the first switching condition in the *n*-th PWM switching interval.

$$U_q(t_o + \Delta t_{\gamma n}) = U_s(t_{\gamma n}). \tag{9}$$

For small increments ΔU_q, the equation can be approximated with respect to the system's initial state using the first two terms of the Taylor series. In this case, the left-hand and right-hand sides of the equation take the following form:

$$\begin{aligned} U_q(t_o + \Delta t_{\gamma n}) &= U_q(t_o) + \Delta U_q(t_o) + \frac{dU_q(t_o)}{dt}\cdot\Delta t_{\gamma n}; \\ U_s(t) &= U_s(t_o) + \frac{dU_s(t_o)}{dt}\cdot\Delta t_{\gamma n}. \end{aligned} \tag{10}$$

In a closed structure, the control action U_q of a PWM converter contains two components

$$U_q(t_n) = U_{qo}(t_n) + U_p(t_n). \tag{11}$$

The first component is a reference input proportional to the smoothed component, while the second is proportional to the pulsating component contained in the output signal of the equivalent continuous part of the system.

Substituting expression (10), which accounts for the components of the control input defined in (11), into the first switching condition (2), we obtain the following:

$$U_{qo}\left(t_o\right)+\Delta U_{qo}\left(t_o\right)+\frac{dU_{qo}\left(t_o\right)}{dt}\Delta t_{\gamma n}+U_p\left(t_o\right)+\frac{dU_p\left(t_o\right)}{dt}=U_s\left(t_o\right)+\frac{dU_s\left(t_o\right)}{dt}\Delta U_{qn}. \quad (12)$$

From expression (12) we obtain

$$\Delta t_{\gamma n}=\frac{U_{qo}\left(t_o\right)+U_p\left(t_o\right)-U_s\left(t_o\right)+\Delta U_q\left(t_o\right)}{\frac{dU_{qo}\left(t_o\right)}{dt}+\frac{dU_p\left(t_o\right)}{dt}-\frac{dU_s\left(t_o\right)}{dt}}. \quad (13)$$

In expression (13) for deviations of the angular coordinate $\Delta t_{\gamma n}$, the sum of the components $U_q(t_o)$ and $U_s(t_o)$ determines the value of the control action $U_q(t_o)$ at the moment of system equilibrium t_o. From the switching condition (3) it follows that

$$U_{qo}\left(t_o\right)+U_p\left(t_o\right)-U_s\left(t_o\right)=0. \quad (14)$$

Let us perform the limit transition in expression (13) taking into account (14)

$$\frac{dt_{\gamma n}}{dU_q}=\frac{1}{\frac{dU_{qo}\left(t_o\right)}{dt}+\frac{dU_p\left(t_o\right)}{dt}-\frac{dU_s\left(t_o\right)}{dt}}. \quad (15)$$

When the transfer function $W(p)$ of the equivalent continuous part of the closed-loop automatic control system with PWM has a difference between the orders of the denominator and numerator polynomials less than two, then the control input U_q has discontinuities at the switching moments $t_{\gamma n}$. In this case, the derivative of $U_q(t_{\gamma n})$ in equation (15) is equal to its left-hand value.

Introducing the ripple factor notation into equation (15) yields the following expression [15]:

$$F^{-1}=1-\frac{\frac{dU_{qo}\left(t_o\right)}{dt}}{\frac{dU_s\left(t_o\right)}{dt}}-\frac{\frac{dU_p\left(t_o\right)}{dt}}{\frac{dU_s\left(t_o\right)}{dt}}. \quad (16)$$

Thus, we obtain the following expression

$$\frac{dt_{\gamma n}}{dU_q}=-F_n\cdot\frac{dt}{dU_s\left(t_o\right)}. \quad (17)$$

Based on expressions (16) and (17), it can be concluded that in pulse-width converter with PWM the pulsating component affects the pulsation factor.

Taking (17) into account, the expression for $\Delta e_v(t_n)$ can be written as follows.

$$\Delta e_v\left(t_n\right)=U_m\cdot\Delta U_q\left(t_{\gamma n}\right)\cdot F_n\cdot\frac{dt}{dU_s\left(t_o\right)}\cdot\sum_{n=0}^{k_o}\delta\left(t-t_n\right). \quad (18)$$

Analyzing expression (18), it can be stated that for small values of $\Delta U_q(t_{\gamma n})$, the pulse-width converter with PWM acts as a second-order amplitude-pulse modulator, where the gain is determined by the form of the reference signal. The reference signal $U_s(t)$ in the case of one-sided PWM is obtained by integrating the constant voltage U_o.

$$U_s\left(t\right)=\frac{1}{T_i}\cdot\int_{nT}^{nT+t}U_o\cdot dt, \quad (19)$$

where T_i – constant integrator time.

Transforming expression (19) and substituting $T_i = T$, we obtain

$$U_s\left(t\right)=U_o\cdot\frac{t_{\gamma n}}{T}, \quad (20)$$

where from

$$\frac{dt}{dU_s\left(t\right)}=\frac{T}{U_o}. \quad (21)$$

Substituting (21) into (18) and considering the repetition of processes over the pulse-width modulated converter discrete intervals, we obtain

$$\Delta e_v\left(t_{\gamma n}\right)=\Delta U_q\left(t_{\gamma n}\right)\cdot T\cdot\sum_{n=1}^{\infty}K\cdot F\cdot\delta\left(t-t_{\gamma n}\right), \quad (22)$$

where K – PWM transmission coefficient, $K=U_m/U_o$;

T – PWM discrete period.

Fig. 3 shows the pulse model of the converter with PWM constructed according to expression (22).

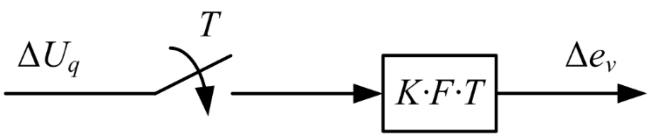

Fig. 3. Pulse model of a pulse-width modulated converter with a one-sided reference signal

This model contains an ideal pulse element with a quantization period T and a reduced part that carries information about the static K and dynamic F transfer coefficients.

Pulse model of a pulse converter with a two-way reference signal. Unlike a converter with one-sided modulation, in this system, two information components are formed at each clock interval, as shown in Fig. 4.

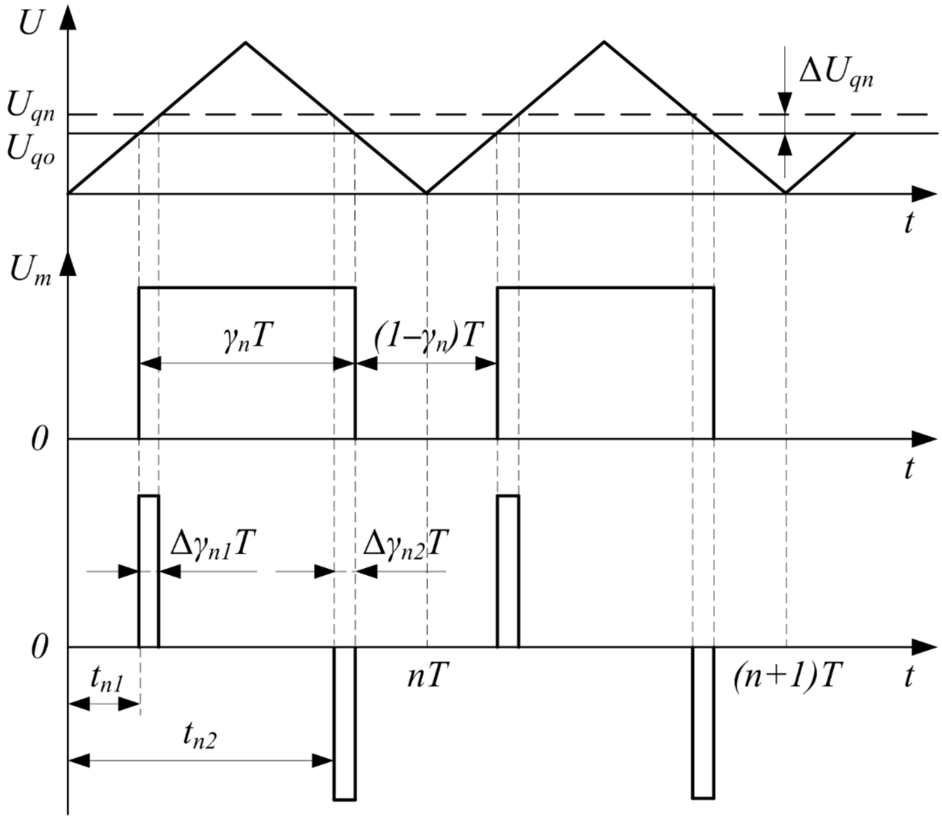

Fig. 4. Voltage diagrams of the pulse-width modulated converter with a two-sided reference signal

In the continuous current mode in the load circuit, the switching condition is determined by the system of equations

$$\begin{aligned} U_s(t_{n1}) &= U_q(t_{n1}); \\ U_s(t_{n2}) &= U_q(t_{n2}). \end{aligned} \tag{23}$$

When switching to intermittent current mode, the first and second conditions are supplemented by a third

$$I_v(t_n; t_n + \tau) = 0, \tag{24}$$

where τ – determines the moment when the load current drops to zero.

From expressions (23) and (24) follows a more complex nature of the flow of dynamic processes in a converter with two-way pulse-width modulation compared to one-way. This is explained by the presence of two meeting points of the control signal and the reference signal.

The increment of the converter output signal under the action of ΔU_q in a closed structure is determined by the transcendental equation with respect to the control signal

$$\Delta e_v\left[\Delta U_q(t_{n1};t_{n2})\right] = e_v\left(U_{qo} + \Delta U_q\right) - e_q\left(U_{qo}\right). \tag{25}$$

The resulting increment of the converter output signal is two periodically repeating sequences of pulses Δt_{n1} and Δt_{n2}.

The dynamic relationship between the input ΔU_q and output Δe_v quantities is determined by linearizing equation (25) relative to the new state of the system at $U_q = U_{qn}$.

$$\Delta e_v = \frac{De_v(t_{n1})}{dU_q}\cdot\frac{\Delta U_q}{1!} + \frac{De_v(t_{n2})}{dU_q}\cdot\frac{\Delta U_q}{1!}. \tag{26}$$

Taking into account the generalized derivative and the amplitudes of the jumps at the switching moments, we obtain

$$\Delta e_v(t_n) = -\frac{\Delta U_q(t_{n1})}{1!}\cdot U_m\cdot\frac{dt_{\gamma n1}}{dU_q}\cdot\sum_{n=o}^{k_o}\partial(t-t_{n1}) -$$

$$-\frac{\Delta U_q(t_{n2})}{1!}\cdot U_m\cdot\frac{dt_{\gamma n2}}{dU_q}\cdot\sum_{n=o}^{k_o}\partial(t-t_{n2}). \tag{27}$$

The time coordinates $t_{\gamma n1}$ and $t_{\gamma n2}$ are determined from the switching conditions (23). For a closed-loop structure containing a converter with two-sided pulse-width modulation, the switching conditions are as follows:

– for the first meeting point

$$\Delta U_q(t_{n1}+t_{\gamma n1}) = U_s(t_{n1}); \tag{28}$$

– for the second meeting point

$$\Delta U_q(t_{n2}+t_{\gamma n2}) = U_s(t_{n2}). \tag{29}$$

For small increments $\Delta t_{\gamma n1}$ and $\Delta t_{\gamma n2}$, transcendental equations (28) and (29) can be approximated relative to the initial state of the system by the first two terms of the Taylor series. In this case, the left and right parts of these equations, taking into account that in a closed structure the control signal contains information about the pulsating component of the output voltage, take the form:

– for the first meeting point

$$U_{qo}(t_{n1}) + \Delta U_{qo}(t_{n1}) + \frac{dU_{qo}(t_{n1})}{dt}\Delta t_{\gamma n1} + U_p(t_{n1}) + \frac{dU_p(t_{n1})}{dt}\Delta t_{\gamma n1} =$$

$$= U_s(t_{n1}) + \frac{dU_s(t_{n1})}{dt}\Delta t_{\gamma n1}; \tag{30}$$

– for the second meeting point

$$U_{qo}(t_{n2}) + \Delta U_{qo}(t_{n2}) + \frac{dU_{qo}(t_{n2})}{dt}\Delta t_{\gamma n2} + U_p(t_{n2}) + \frac{dU_p(t_{n2})}{dt}\Delta t_{\gamma n2} =$$

$$= U_s\left(t_{n2}\right) + \frac{dU_s\left(t_{n2}\right)}{dt}\Delta t_{\gamma n2}. \qquad (31)$$

From expressions (30) and (31) it is easy to obtain expressions establishing the relationship between the increments of the reference signal ΔU_{qo} and the durations of the increments $\Delta t_{\gamma n}$ of the output pulse of the pulse-width controller for both meeting points:

$$\Delta t_{\gamma n1} = \frac{U_{qo}\left(t_{n1}\right) + U_p\left(t_{n1}\right) - U_s\left(t_{n1}\right) + U_{qo}\left(t_{n1}\right)}{\frac{dU_s\left(t_{n1}\right)}{dt} - \frac{dU_{qo}\left(t_{n1}\right)}{dt} - \frac{dU_p\left(t_{n1}\right)}{dt}}; \qquad (32)$$

$$\Delta t_{\gamma n2} = \frac{U_{qo}\left(t_{n2}\right) + U_p\left(t_{n2}\right) - U_s\left(t_{n2}\right) + U_{qo}\left(t_{n2}\right)}{\frac{dU_s\left(t_{n2}\right)}{dt} - \frac{dU_{qo}\left(t_{n2}\right)}{dt} - \frac{dU_p\left(t_{n2}\right)}{dt}}. \qquad (33)$$

In steady state, in accordance with the switching condition

$$U_{qo}\left(t_{n1}\right) + U_p\left(t_{n1}\right) - U_s\left(t_{n1}\right) = 0; \qquad (34)$$

$$U_{qo}\left(t_{n2}\right) + U_p\left(t_{n2}\right) - U_s\left(t_{n2}\right) = 0. \qquad (35)$$

As a result of limit transitions in expressions (32) and (33), taking into account (34) and (35), we obtain

$$\frac{\Delta t_{\gamma n1}}{dU_{qo}\left(t_{n1}\right)} = \frac{1}{\frac{dU_s\left(t_{n1}\right)}{dt} - \frac{dU_{qo}\left(t_{n1}\right)}{dt} - \frac{dU_p\left(t_{n1}\right)}{dt}}; \qquad (36)$$

$$\frac{\Delta t_{\gamma n2}}{dU_{qo}\left(t_{n2}\right)} = \frac{1}{\frac{dU_s\left(t_{n2}\right)}{dt} - \frac{dU_{qo}\left(t_{n2}\right)}{dt} - \frac{dU_p\left(t_{n2}\right)}{dt}}. \qquad (37)$$

By introducing the pulsation factor designation into expressions (36) and (37)

$$F_1^{-1} = 1 - \frac{\frac{dU_{qo}\left(t_{n1}\right)}{dt}}{\frac{dU_s\left(t_{n1}\right)}{dt}} - \frac{\frac{dU_p\left(t_{n1}\right)}{dt}}{\frac{dU_s\left(t_{n1}\right)}{dt}}; \qquad (38)$$

$$F_2^{-1} = 1 - \frac{\frac{dU_{qo}(t_{n2})}{dt}}{\frac{dU_s(t_{n2})}{dt}} - \frac{\frac{dU_p(t_{n2})}{dt}}{\frac{dU_s(t_{n2})}{dt}}, \tag{39}$$

we get

$$\frac{dt_{\gamma n1}}{dU_{qo}(t_{n1})} = -F_1 \frac{dt}{dU_s(t_{n1})}; \tag{40}$$

$$\frac{dt_{\gamma n2}}{dU_{qo}(t_{n2})} = -F_2 \frac{dt}{dU_s(t_{n2})}. \tag{41}$$

Taking into account expressions (40), (41) and the static transfer coefficient *K*, the expression for the increment of the output signal of the pulse-width modulated converter takes the form

$$\Delta e_v(t_n) = \Delta U_q(t_{n1}) T \sum_{n=1}^{\infty} K F_1 \delta(t - t_{n1}) - \Delta U_q(t_{n2}) T \sum_{n=1}^{\infty} K F_2 \delta(t - t_{n2}). \tag{42}$$

From expression (42) it follows that for small values of the control signal increment, a pulse converter with two-way pulse-width modulation is a second-order amplitude-pulse modulator (Fig. 5), in which information is transmitted by two channels.

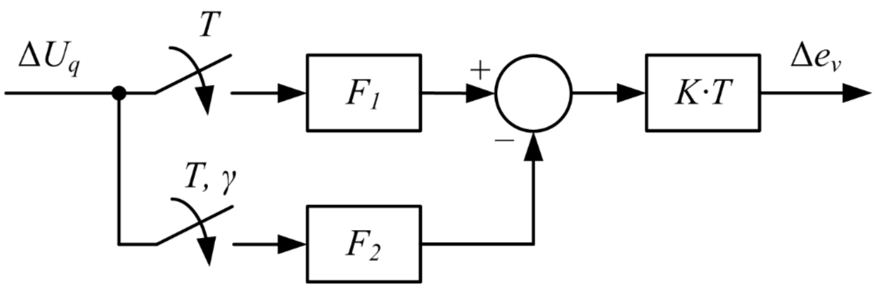

Fig. 5. Pulse model of a pulse-width modulated converter with a two-sided reference signal

The resulting model enables the study of dynamic processes in automatic control systems with a second-order PWM pulsed converter, taking into account discreteness in the continuous current mode.

Automatic voltage regulation system of a pulse-width converter with two-way PWM. To demonstrate the application of the developed pulse models, we will consider a system of automatic control of the anchor current of a braked DC motor with independent excitation [12, 13]. This mode is used in the development of a system of subordinate control of the speed of an electric motor. Electric motor parameters: supply voltage U_D = 220 V, armature current I_a = 46 A, anchor chain resistance R_a = 0,29 Om, armature inductance $L_a = 3{,}5\cdot10^{-3}$ H, shunt resistance $R_1 = 1{,}5\cdot10^{-3}$ Om. Time constant of the anchor chain $T_a = 12\cdot10^{-3}$ s.

The structural diagram of the anchor current control system is shown in Fig. 6.

The transfer function of the reduced continuous part is determined by the transfer functions of the current regulator and the anchor part

$$W(p)=H_{Ia}(p)\cdot H_I(p). \qquad (43)$$

The transfer function of the anchor chain is determined by the expression

$$H_a(p)=\frac{R_1}{R_a}\cdot\frac{1}{T_a\cdot p+1}. \qquad (44)$$

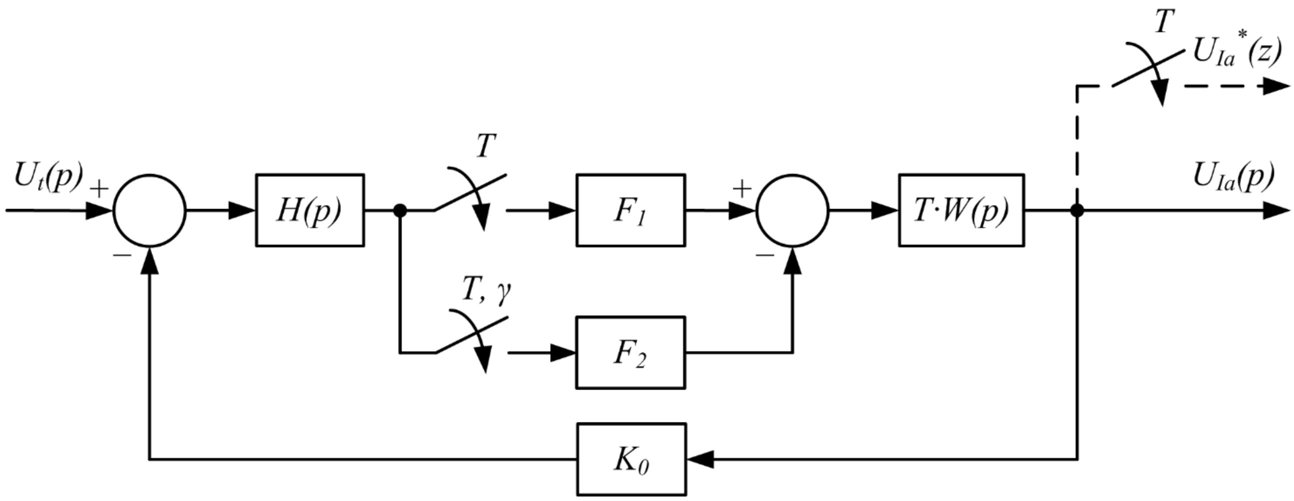

Fig. 6. Structural diagram of the automatic control system of the anchor current of the electric motor

In accordance with [11], to obtain physically feasible conditions for a process of finite duration and first-order astatism, the transfer function of the controller has the form

$$H_I(p)=\frac{K_1\cdot(T_2\cdot p+1)}{T_3\cdot p}, \qquad (45)$$

where $K_1=\dfrac{R_a}{R_1}$.

Then, equation (43), taking into account (44) and (45), takes the form

$$W(p)=\frac{K_0\cdot(T_2\cdot p+1)}{T_3\cdot p\cdot(T_a\cdot p+1)}, \qquad (46)$$

where K_0 - is the static transfer coefficient of the pulse converter.

In accordance with [14, 16] and taking into account that the delay of the pulse element *Tγ*, relative to the pulse element *T*, is less than one clock interval, the relationship between the input and output signals is determined by the expression

$$U_{Ia}^{*}\left(z,\varepsilon\right)\Big|_{\varepsilon=0}=\frac{K_0\cdot T\cdot\left[U_tH^{*}\left(z,0\right)\cdot F_1+U_tH^{*}\left(z,1-\gamma\right)\cdot F_2\right]\cdot W^{*}\left(z\right)}{1+z^{-1}\cdot K_0\cdot T\cdot\left[W^{*}\left(z,1\right)\cdot F_1+W^{*}\left(z,1-\gamma\right)\cdot F_2\right]}. \qquad (47)$$

By performing a modified Z-transformation of the transfer function (46) in accordance with [16], we obtain the characteristic equation

$$1+\frac{T}{T_3}\cdot\left\{\left[\frac{1}{z-1}-\left(1-\frac{T_3}{T_a}\right)\cdot\frac{e^{-\frac{T}{T_a}}}{z-e^{-\frac{T}{T_a}}}\right]\cdot F_1+\left[\frac{1}{z-1}-\left(1-\frac{T_3}{T_a}\right)\cdot\frac{e^{-(1-\gamma)\cdot\frac{T}{T_a}}}{z-e^{-\frac{T}{T_a}}}\right]\cdot F_2\right\}=0. \qquad (48)$$

In accordance with [15, 17, 18], we write expressions for the pulsation factors

$$F_1^{-1}=1+\frac{T}{2\cdot T_3}\cdot\left[1-\gamma+\left(\frac{T_2}{T_a}-1\right)\cdot\frac{e^{-\gamma\cdot\frac{T}{T_a}}-e^{-\frac{T}{T_a}}}{1-e^{-\frac{T}{T_a}}}\right]; \qquad (49)$$

$$F_2^{-1}=1+\frac{T}{2\cdot T_3}\cdot\left[\gamma+\left(\frac{T_2}{T_a}-1\right)\cdot\frac{e^{-(1-\gamma)\cdot\frac{T}{T_a}}-e^{-\frac{T}{T_a}}}{1-e^{-\frac{T}{T_a}}}\right]. \qquad (50)$$

Analysis of expressions (49) and (50) shows that at $\gamma = 0,5$ the pulsation factors are equal to each other. Then the conditions for adjusting the control system to a finite duration process have the form [13, 14]

$$T_2=T_a\cdot\frac{e^{-\frac{T}{T_a}}\cdot\left(2-e^{-\frac{T}{T_a}}\right)+e^{-\frac{T}{2\cdot T_a}}}{e^{-\frac{T}{T_a}}+e^{-\frac{T}{2\cdot T_a}}}; \qquad (51)$$

$$T_3=T\cdot\left(2-e^{-\frac{T}{T_a}}\right)\cdot\left[\frac{1}{2}+\left(\frac{T_2}{T_a}-1\right)\cdot\frac{e^{-\frac{T}{2\cdot T_a}}-e^{-\frac{T}{T_a}}}{1-e^{-\frac{T}{T_a}}}\right]. \qquad (52)$$

For the given parameters of the anchor circuit of the electric motor, the values of the time constants of the current regulator are obtained: the differentiating part – $T_2 = 12,45\cdot10^{-3}$ s; integrating part – T_3=$0,56\cdot10^{-3}$ s.

For experimental studies of the considered system of automatic regulation of the anchor current of a braked electric motor, a simulation model was created in the MATLAB environment, presented in Fig. 7.

The input circuit of the model consists of a voltage source DCV, its internal resistance R1 and capacitor C. The converter contains a power key, implemented on the IGBT1 transistor, and a return diode Diode. The anchor circuit of the electric motor on the model is presented as a series RL connection, designated as R, and a current shunt on the resistor R2. The converter control system contains a reference signal generator GPN and a comparator. The automatic anchor current control system has a source of the reference signal $U_q = U_y$, a source of input action Stap1 current regulator Transfer Fcn1 and a feedback link Transfer Fcn2. For visual study of transient processes, an oscilloscope Skope1 is used.

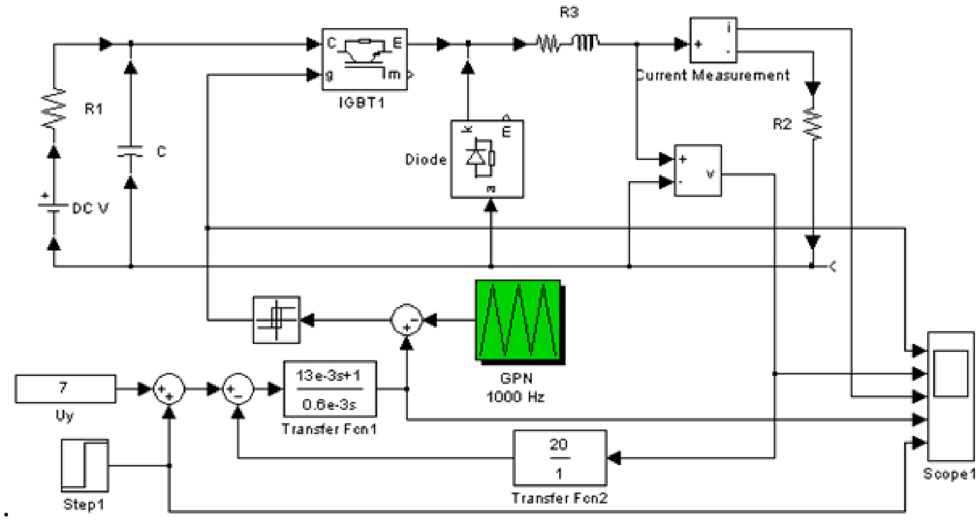

Fig. 7. Simulation model of the automatic control system of anchor current

Fig. 8 shows oscillograms of transient processes of the anchor current control system configured for the conditions of a finite duration process.

Analysis of oscillograms shows that the transient process ends in two clock intervals of the pulse voltage converter, which corresponds to the order of the system.

Analysis of the dynamic processes of the considered system confirms the correctness of the analytical results obtained in the article.

Conclusions. On the basis of the conducted research, the following conclusions can be drawn:

– the effectiveness of using the generalized derivative method for obtaining pulse models of a pulsed DC voltage converter with pulse-width modulation in continuous current mode has been demonstrated;

– the obtained pulse models make it possible to: accurately describe the dynamic processes in automatic control systems with a pulsed DC voltage convert-er; calculate regulator parameters to achieve maximum response speed;

– the practical significance of the theoretical principles presented in the article is confirmed by the results of a study conducted using a simulation model in the MATLAB environment, representing an automatic armature current control system of an electric motor configured for a finite-duration process.

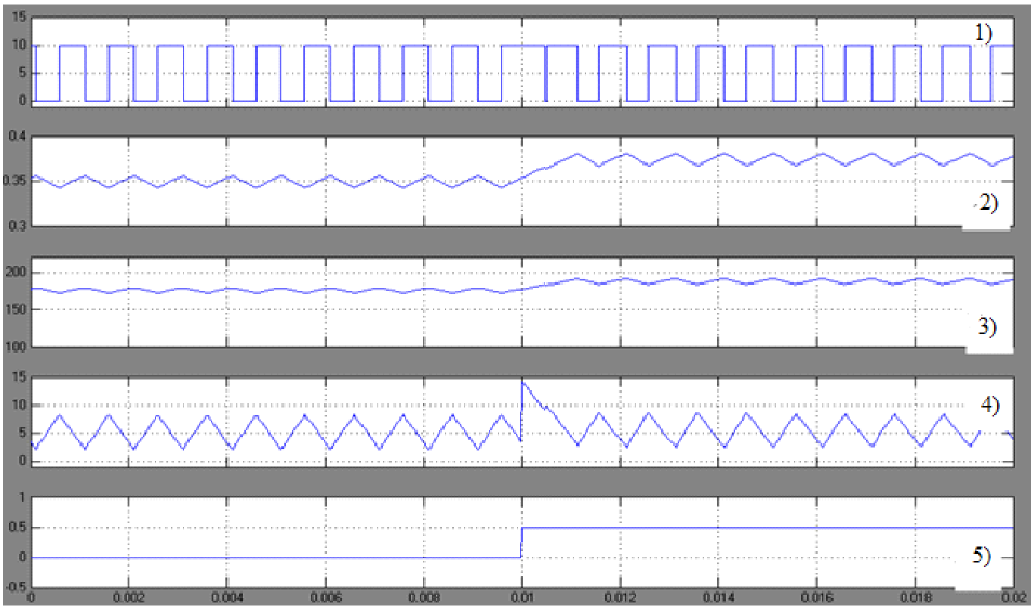

Fig. 8. Oscillograms of transient processes:
1 – IGBT transistor control signal; *2* – shunt voltage R3; *3* – armature current of the electric motor; *4* – output signal of the current regulator Transfer Fcn1; *5* – external disturbance

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